

# MATHEMATICS

## Chapter 4: Practical Geometry



## Practical Geometry

1. A quadrilateral is a four-sided polygon.
2. A quadrilateral has four sides, four angles and two diagonals, i.e., 10 elements. A quadrilateral can be constructed uniquely if at least five of its elements are given.
3. A quadrilateral can be constructed uniquely, if we know any one of the following:
  - i. Four sides and one diagonal
  - ii. Four sides and one angle
  - iii. Two diagonals and three sides
  - iv. Two adjacent sides and three angles
  - v. Three sides and two included angles
4. For the construction of different type of quadrilaterals like parallelogram, rhombus, trapezium etc. we use their properties.

### Construction of a Quadrilateral

It is very easy to construct a quadrilateral when its five measurements are determined that is

- The length of the four sides and the length of its diagonal is known
- The length of the three sides and the length of the two diagonals are known
- If the three angles and two adjacent sides are given
- If the three sides and two angles are given

#### 4 Sides and 1 Diagonal

Construction of a Quadrilateral when different measures of sides and angles are given

A unique quadrilateral can be constructed when the following measurements are given:

- Four sides and one diagonal.
- Two diagonals and three sides.
- Two adjacent sides and three angles.
- Three sides and two included angles.
- When other special properties are known.

#### SSS Construction

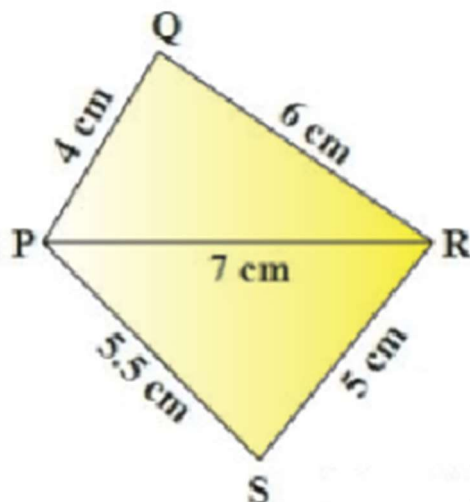
- To construct a  $\triangle ABC$ , the length of whose sides are,  $AB = x$  cm,  $BC = y$  cm, and  $AC = z$  cm, we will do it in the following manner:
- Construct a line segment  $AB$ , whose length is  $x$  cm.
- With  $A$  as the center, draw an arc of radius  $z$  cm.
- With  $B$  as the center, draw an arc of radius  $y$  cm on the same side. The point where the arcs intersect is the required point  $C$ .
- Join  $AC$  and  $BC$ .

$\triangle ABC$  is the required triangle.

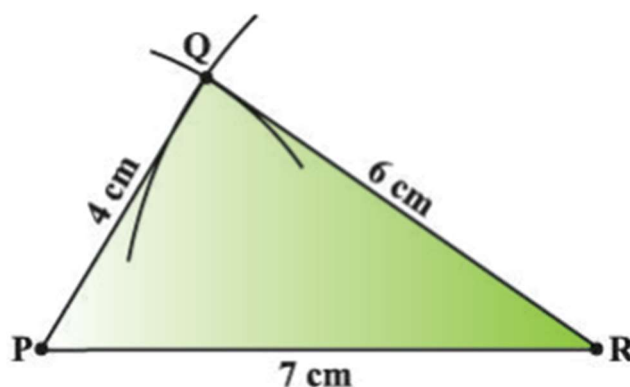
### Construction of a Quadrilateral when four sides and one diagonal are given

Suppose we have to construct a quadrilateral PQRS, where  $PQ = 4\text{ cm}$ ,  $QR = 6\text{ cm}$ ,  $RS = 5\text{ cm}$ ,  $PS = 5.5\text{ cm}$  and  $PR = 7\text{ cm}$ .

**Step 1:** Draw a rough sketch to visualize the quadrilateral.



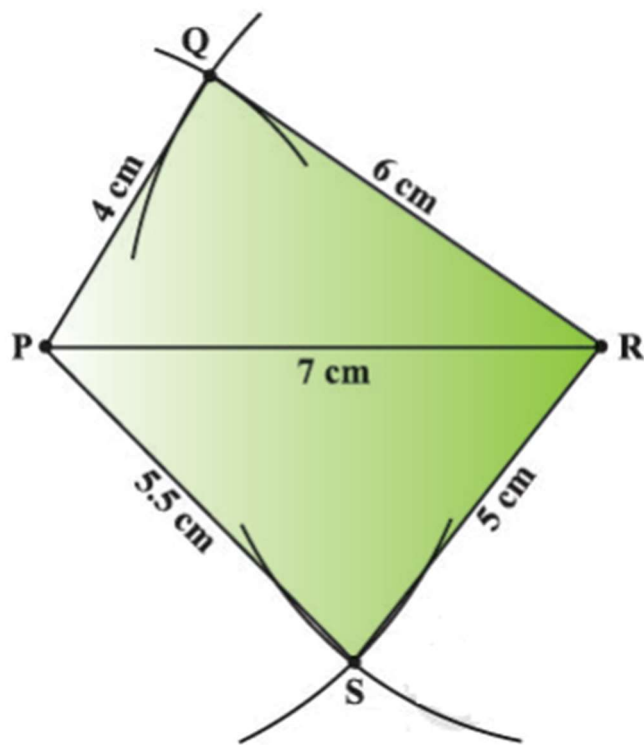
**Step 2:** Draw  $\triangle PQR$  as it can be constructed using SSS construction condition.



**Step 3:** Now we have to locate S, which is at a distance of 5.5 cm from P and 5 cm from R. Also it will be on the opposite side of Q.

With P as center draw an arc of radius 5.5 cm. With R as center draw an arc of radius 5 cm. S is the point of intersection of the two arcs.

**Step 4:** Join PS and RS. PQRS is the required quadrilateral.

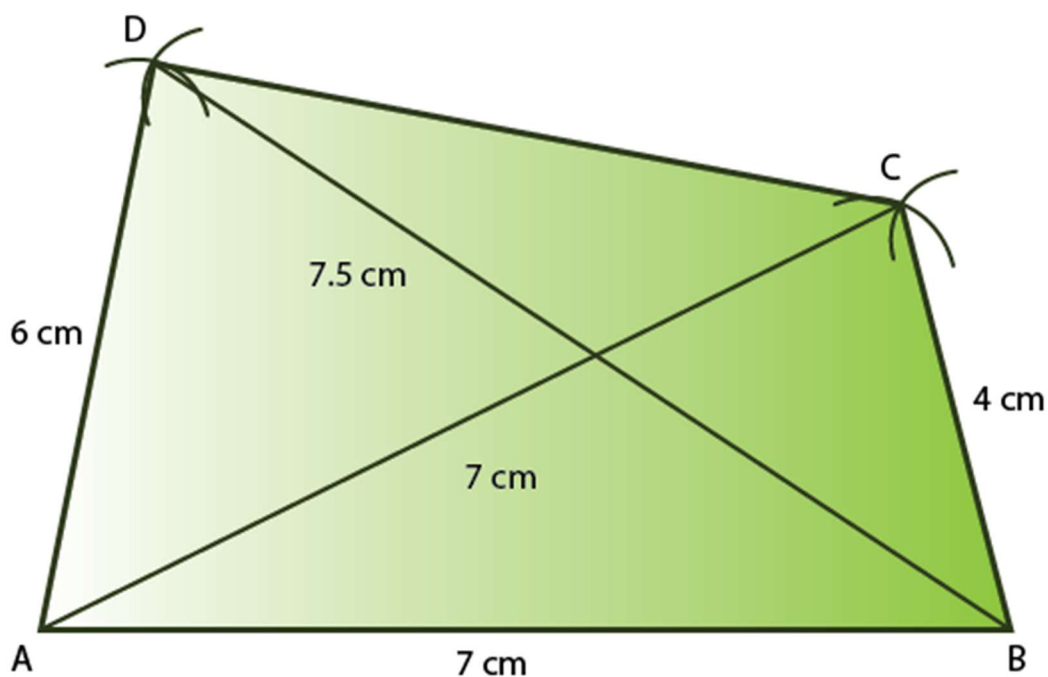


### 3 Sides and 2 Diagonals

Construction of a Quadrilateral when two diagonals and three sides are given

Construct a quadrilateral ABCD given, AB = 7 cm, AD = 6 cm, AC = 7 cm, BD = 7.5 cm and BC = 4 cm.

[Make a rough figure for your reference]



**Steps of construction of the quadrilateral:**

**Step 1:**  $\triangle ABC$  can be drawn by SSS construction condition since all its sides are known.

**Step 2:** With A as center and radius 6 cm (AD), draw an arc.

**Step 3:** With B as center and radius 7.5 cm (BD) draw another arc to cut the previous arc at D

**Step 4:** Join AD, BD, and CD.

ABCD is the required quadrilateral

## 2 Adjacent Sides and 3 Angles

### Construction of a Quadrilateral when two adjacent sides and three angles are given

Construct a quadrilateral ALPN, where  $AL = 6.5$  cm,  $LP = 4$  cm,  $\angle NAL = 110^\circ$ ,  $\angle ALP = 75^\circ$  and  $\angle LPN = 90^\circ$ .

[Draw a rough Sketch for your reference]:

Steps of construction of the quadrilateral:

Step 1: Draw the line segment AL of length 6.5 cm.

Step 2: Make  $\angle ALY = 75^\circ$  at L.

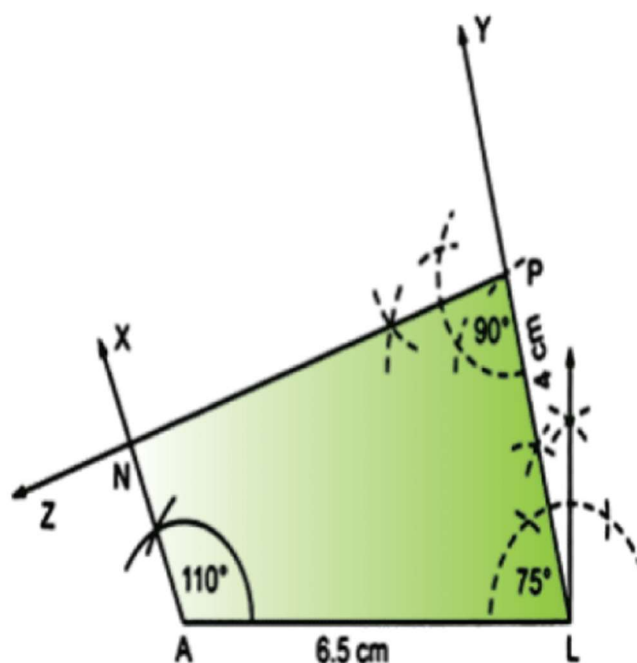
Step 3: Make  $\angle LAX = 110^\circ$  at A.

Step 4: With L as center and radius equal to 4 cm, cut an arc on the ray LY at P.

Step 5: Make  $\angle LPZ = 90^\circ$  at P.

Step 6: Name the point of intersection of rays PZ and AX as N.

ALPN is the required quadrilateral.

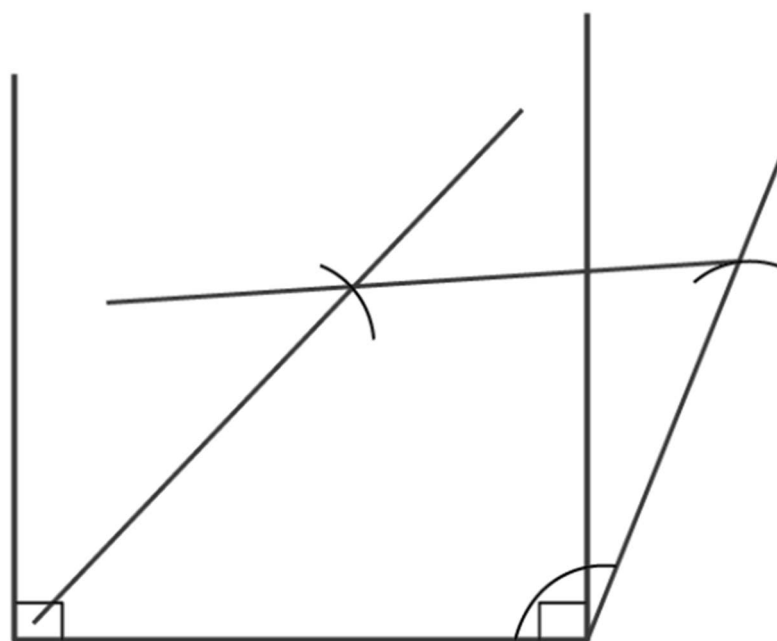


## 3 Sides and 2 Included Angles

### Construction of a Quadrilateral when three Sides and two included angles are given

Construct a quadrilateral ABCD, Where  $AB = 4.5$  cm;  $BC = 3.5$  cm,  $CD = 5$  cm  $\angle ABC = 45^\circ$ ,  $\angle BCD = 150^\circ$

[Make a rough figure for your reference]



#### Steps of construction of the quadrilateral:

Step 1: Draw a line segment BC of length 3.5 cm.

Step 2: Make  $\angle LBC = 45^\circ$ .

Step 3: Make  $\angle BCM = 150^\circ$ .

Step 4: With B as center and radius equal to 4.5 cm, cut an arc on the ray LB at A.

Step 5: With C as the center and radius equal to 5 cm, cut an arc on the ray CM at D.

Step 6: Join AD.

ABCD is the required quadrilateral.

#### 4 Sides and One Diagonal are Given

Let us say you are required to construct a quadrilateral PQRS where the measurements are:

PQ = 5 cm

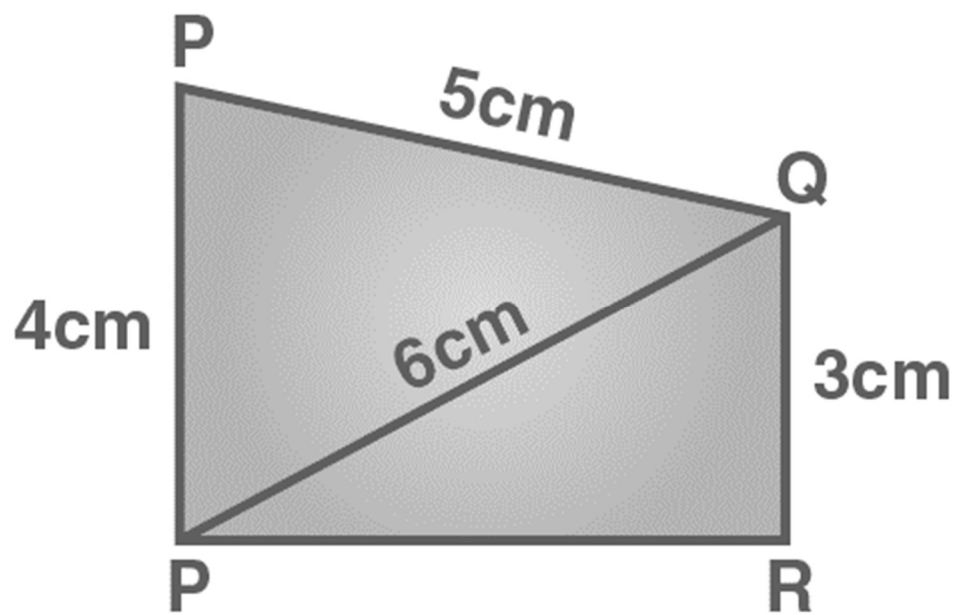
QR = 3 cm

RS = 5 cm

PS = 4 cm

Diagonal SQ = 6 cm

For the construction of quadrilaterals with some of the measurements given, we first draw a rough figure of the quadrilateral with the given dimensions, as shown below.

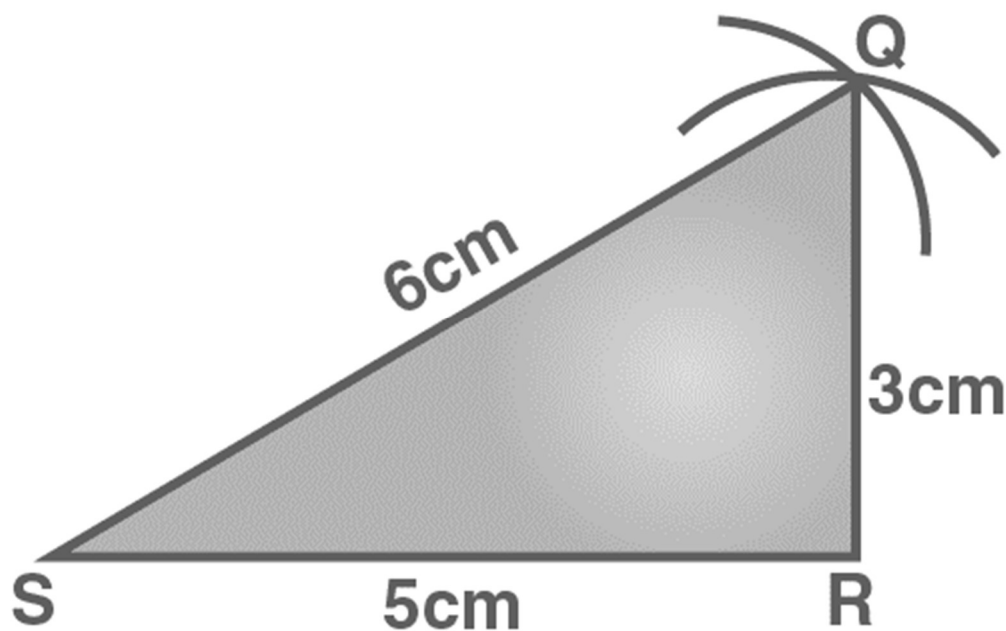


Now starting with the construction, the steps are:

- Draw a line segment of length 5 cm and mark the ends as S and R.

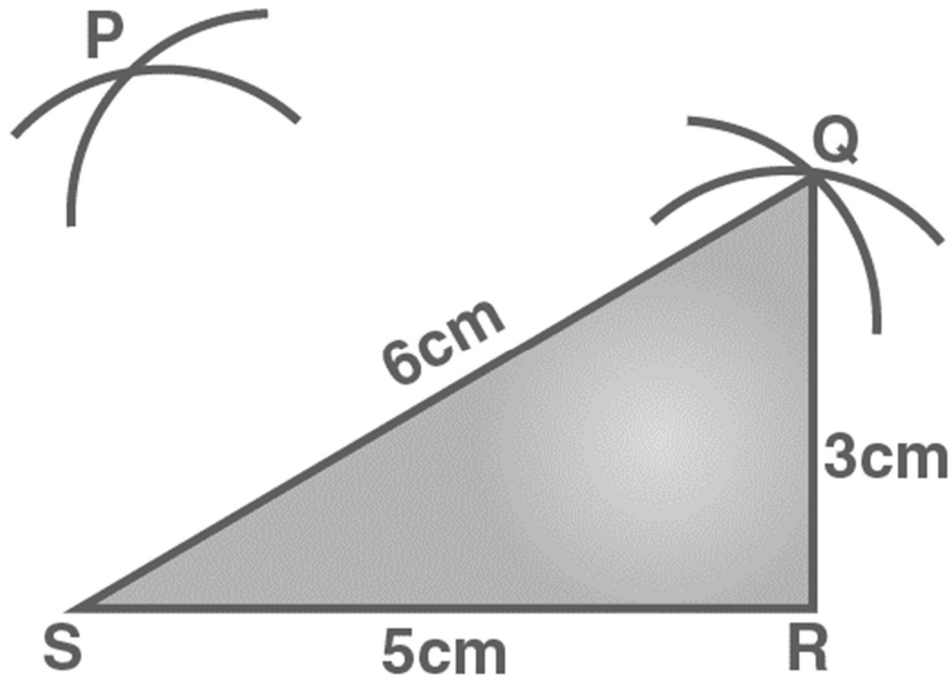


- Set your compass to the radius of 3 cm and make an arc from the point R above the line segment.
- Set the compass to the radius of 6 cm and make an arc from the point S on the previous arc.
- Mark the point as Q where the two arc cross each other. Join the points S and Q as well as R and Q.

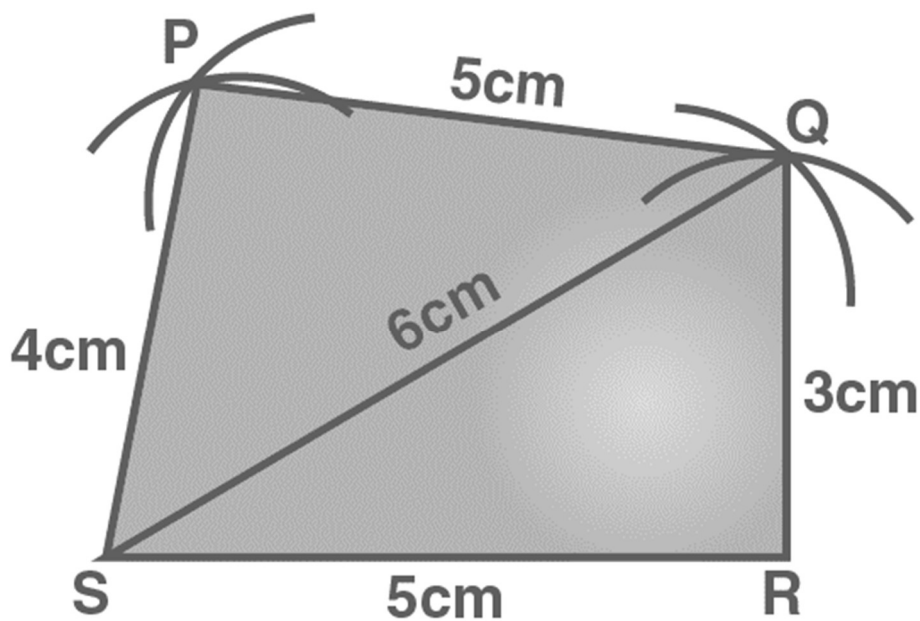


- Set the compass to the radius of 5 cm and make an arc from the point Q.
- Set the compass to the radius of 4 cm and make an arc from the point S on the

previous arc.



- Mark the point as P where the two arc cross each other.
- Join the points P and Q as well as P and S.



- You obtain the quadrilateral PQRS of the required measurements.

### Special Quadrilaterals

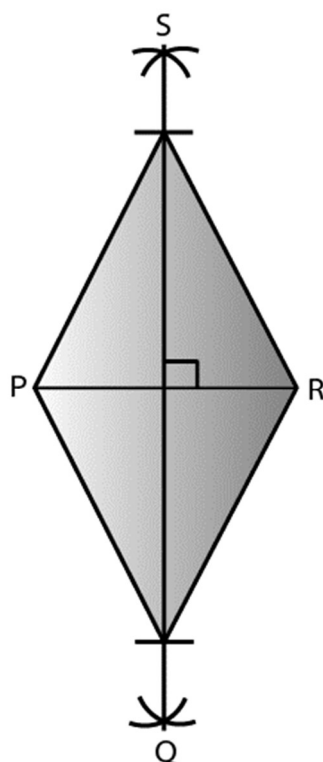
#### Construction of a Quadrilateral When Other Special Properties Are Known

Construct a rhombus PQRS with diagonals  $PR = 5.2$  cm and  $QS = 6.4$  cm

[Make a rough figure for your reference]

**Note:** Diagonals of a rhombus are perpendicular bisectors of each other.





### Steps of construction of the Rhombus:

Step 1: Draw a line segment PR of length 5.2 cm.

Step 2: Draw the perpendicular bisector of PR. Name the point O, where the perpendicular bisector of PR and PR intersect.

Step 3: With O as center and radius equal to 3.2 cm cut arcs on both sides of the perpendicular bisector. Name them as Q and S.

Step 4: Join, PQ, QR, RS, and PS.

### Introduction to Practical Geometry

#### Number of measurements necessary for construction of a unique Quadrilateral

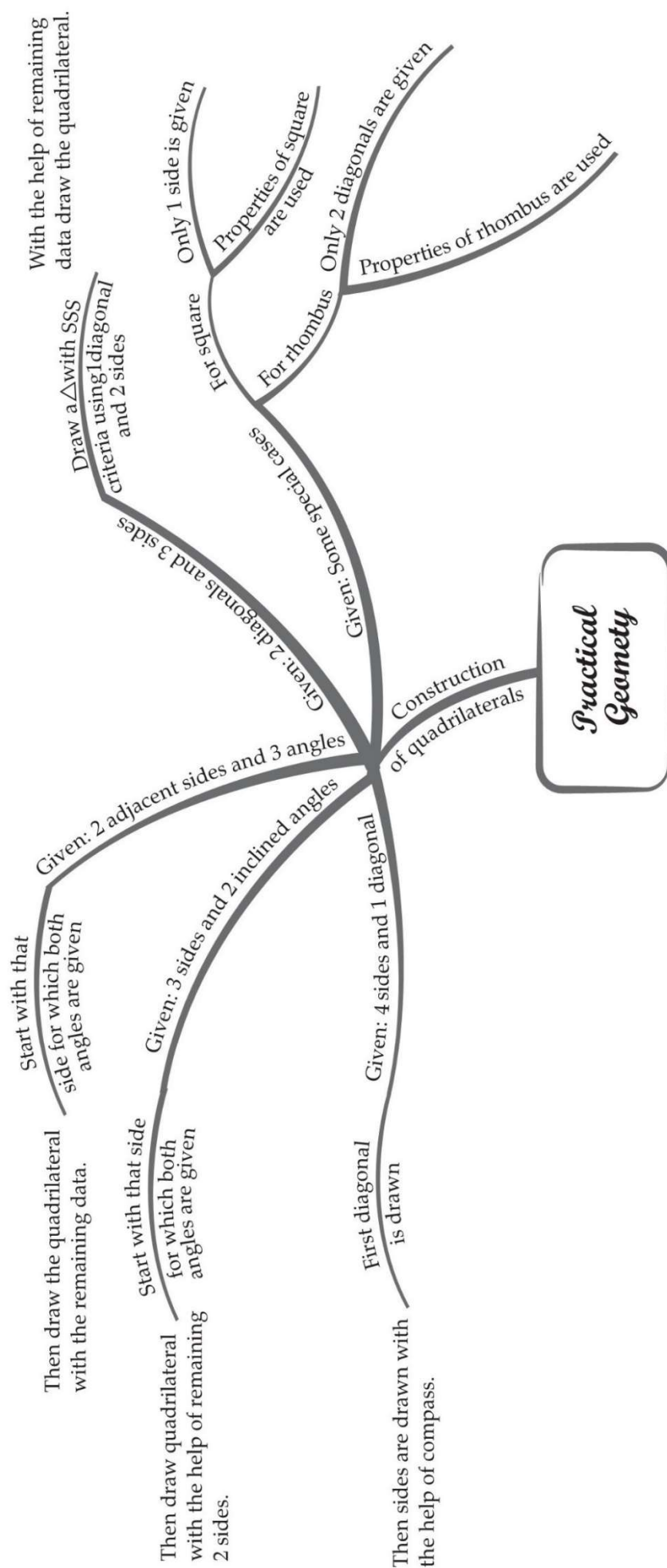
To draw a unique quadrilateral, we need at least five measurements of sides and angles. However, it is not necessary that we will get a unique quadrilateral if we have the measurements of any five combinations of sides and angles.

For example, a unique quadrilateral can be drawn if we are given the measurement of four sides and one diagonal of a quadrilateral.

However, a unique quadrilateral will not be drawn if we are given the measurement of two diagonals and three angles of a quadrilateral.\

# MIND MAP : LEARNING MADE SIMPLE

## CHAPTER-2



## Important Questions

### Multiple Choice Questions-

Question 1. Sum of all interior angles of a polygon with (n) sides is given by

- (a)  $(n - 2) \times 180^\circ$
- (b)  $n - 2 \times 180^\circ$
- (c)  $(n + 2) \times 180^\circ$
- (d)  $(n + 2) \times 180^\circ$

Question 2. Polygons that have no portions of their diagonals in their exteriors are called

- (a) triangles
- (b) convex
- (c) concave
- (d) squares

Question 3. What is the number of sides in Hexagon ?

- (a) 4
- (b) 7
- (c) 6
- (d) 5

Question 4. A parallelogram must be a rectangle if its diagonals

- (a) bisect the angles to which they are drawn
- (b) are perpendicular to each other
- (c) bisect each other
- (d) are congruent

Question 5. Diagonals of a rectangle:

- (a) equal to each other
- (b) not equal
- (c) one is double of the other
- (d) none of these

Question 6. A simple closed curve made up of only \_\_\_\_\_ is called a polygon

- (a) lines
- (b) curves
- (c) closed curves

(d) line segments

Question 7. To construct a quadrilateral uniquely, it is necessary to know at least \_\_\_\_\_ of its parts.

(a) 5

(b) 4

(c) 3

(d) 2

Question 8. All the angles of a regular polygon are of \_\_\_\_\_.

(a)  $90^\circ$

(b)  $60^\circ$

(c) equal length

(d) equal measure

Question 9. The diagonals of a square bisect each other at \_\_\_\_\_ angle.

(a) acute

(b) right

(c) obtuse

(d) reflex

Question 10. The quadrilateral whose diagonals are equal and bisect each other at right angle is \_\_\_\_\_.

(a) Triangle

(b) Square

(c) Rhombus

(d) None of these

### Very Short Questions:

### Short Questions :

### Long Questions :

1. Construct a quadrilateral PQRS, given that  $QR = 4.5$  cm,  $PS = 5.5$  cm,  $RS = 5$  cm and the diagonal  $PR = 5.5$  cm and diagonal  $SQ = 7$  cm.
2. Construct a quadrilateral ABCD in which  $AB = 4$  cm,  $BC = 3.5$  cm,  $CD = 5$  cm,  $AD = 5.5$  cm and  $\angle B = 75^\circ$ .
3. Construct a square whose side is 5 cm.
4. Construct a rhombus ABCD in which  $AB = 5.8$  cm and  $AC = 7.5$  cm.
5. Construct a rhombus whose diagonals are 6 cm and 8 cm.

6. Construct a rectangle whose diagonal is 5 cm and the angle between the diagonal is  $50^\circ$ .
7. Construct a quadrilateral ABCD in which  $BC = 4$  cm,  $\angle B = 60^\circ$ ,  $\angle C = 135^\circ$ ,  $AB = 5$  cm and  $\angle A = 90^\circ$ .
8. Construct a parallelogram ABCD in which  $AB = 5.5$  cm,  $AC = 7$  cm and  $BD = 8$  cm.
9. Construct a rhombus PAIR, given that  $PA = 6$  cm and angle  $\angle A = 110^\circ$ .

### Answer Key-

### Multiple Choice questions-

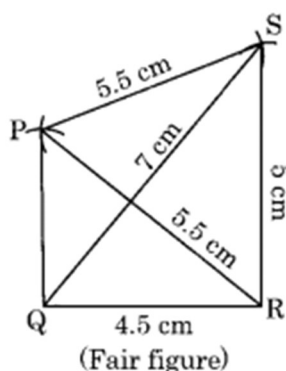
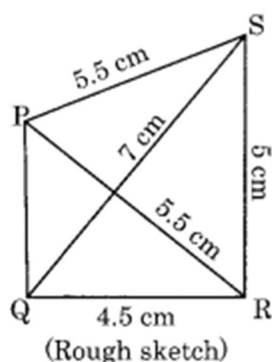
1. (a)  $(n - 2) \times 180^\circ$
2. (b) convex
3. (c) 6
4. (d) are congruent
5. (a) equal to each other
6. (d) line segments
7. (a) 5
8. (d) equal measure
9. (b) right
10. (b) Square

### Very Short Answer :

### Short Answer :

### Long Answer :

1.



Construction:

Step I: Draw  $QR = 4.5$  cm.

Step II: Draw an arc with centre R and radius 5 cm.

Step III: Draw another arc with centre Q and radius 7 cm to meet the previous arc at S.

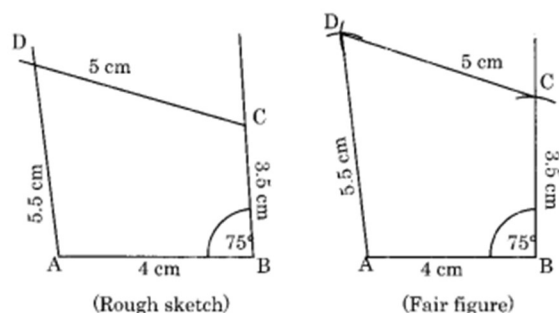
Step IV: Join RS and QS.

Step V: Draw two arcs with centre S and R and radius 5.5 cm each to meet each other at P.

Step VI: Join RP, SP and PQ.

Thus PQRS is the required quadrilateral.

2.



Construction:

Step I: Draw  $AB = 4$  cm.

Step II: Draw an angle of  $75^\circ$  at B and cut  $BC = 3.5$  cm.

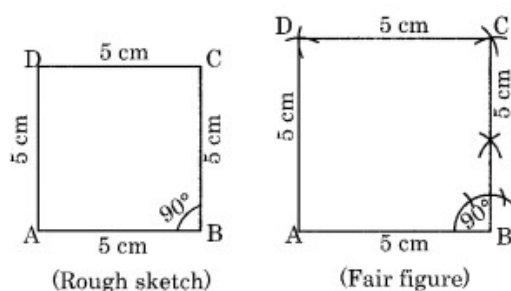
Step III: Draw an arc with centre C and radius 5 cm.

Step IV: Draw another arc with centre A and radius 5.5 cm to meet the previous arc at D.

Step V: Join CD and AD.

Thus ABCD is the required quadrilateral.

3.



Construction:

Step I: Draw  $AB = 5$  cm.

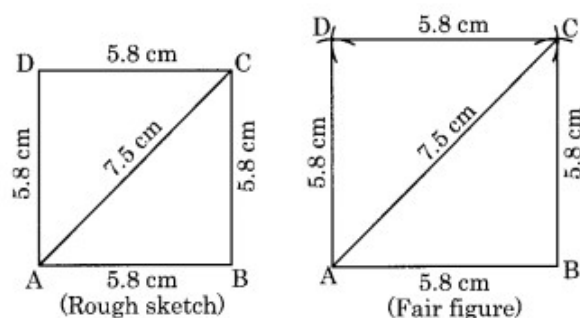
Step II: Draw an angle of  $90^\circ$  at B and cut  $BC = 5$  cm.

Step III: Draw two arcs with centre A and C and same radii of 5 cm which meet each other at D.

Step IV: Join AD and CD.

Thus, ABCD is the required square.

4.



Construction:

Step I: Draw  $AB = 5.8$  cm.

Step II: Draw an arc with centre B and radius 5.8 cm.

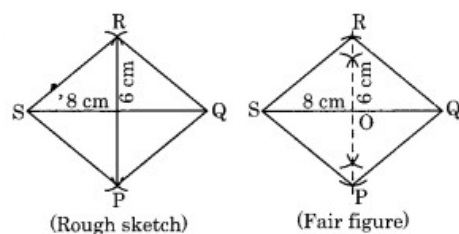
Step III: Draw another arc with centre A and radius 7.5 cm to meet the previous arc at C.

Step IV: Draw two arcs with centres A and C and of the same radius 5.8 cm to meet each other at D.

Step V: Join BC, AC, CD and AD.

Thus ABCD is the required rhombus.

5.



Construction:

Step I: Draw  $SQ = 8$  cm.

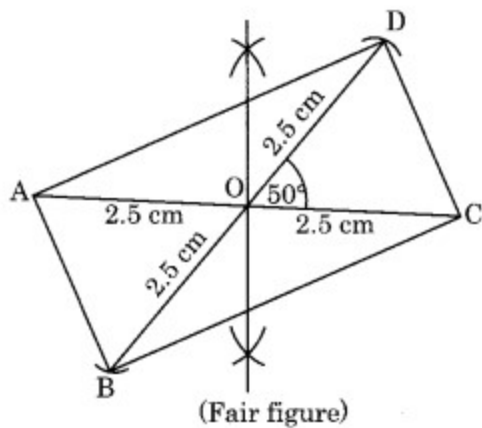
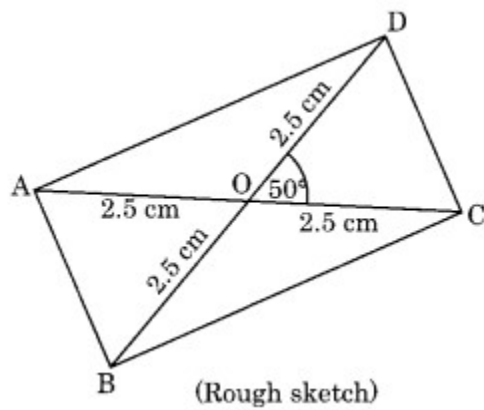
Step II: Draw a right bisector of SQ at O.

Step III: Draw two arcs with centre O and radius 3 cm each to cut the right bisector at P and R.

Step IV: Join PQ, QR, RS and SP.

Thus PQRS is the required rhombus.

6.



Construction:

Step I: Draw  $AC = 5$  cm.

Step II: Draw the right bisector of  $AC$  at  $O$ .

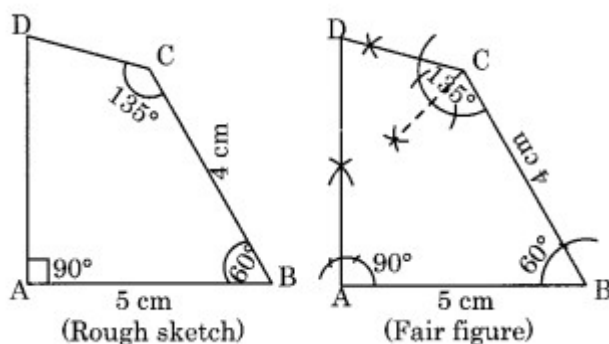
Step III: Draw an angle of  $50^\circ$  at  $O$  and produce both sides.

Step IV: Draw two arcs with centre  $O$  and of the same radius  $2.5$  cm to cut at  $B$  and  $D$ .

Step V: Join  $AB$ ,  $BC$ ,  $CD$  and  $DA$ .

Thus,  $ABCD$  is the required rectangle.

7.



Construction:

Step I: Draw  $AB = 5$  cm.

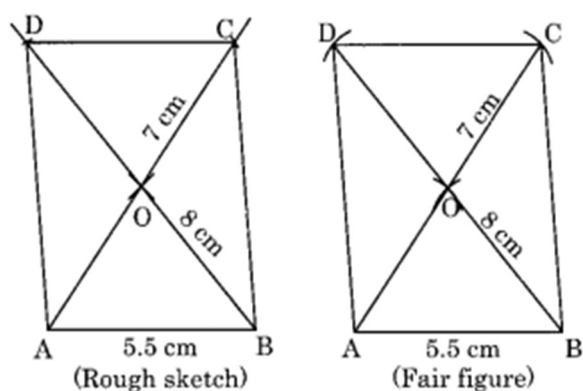
Step II: Draw the angle of  $60^\circ$  at  $B$  and cut  $BC = 4$  cm.



Step III: Draw an angle of  $135^\circ$  at C and angle of  $90^\circ$  at A which meet each other at D.

Thus, ABCD is the required quadrilateral.

8.



Construction:

Step I: Draw  $AB = 5.5$  cm.

Step II: Draw an arc with centre B and radius  $\frac{8}{2}$  cm = 4 cm.

Step III: Draw another arc with centre A and radius  $\frac{7}{2}$  cm = 3.5 cm which cuts the previous arc at O.

Step IV: Join AO and produce to C such that  $AO = OC$ .

Step V: Join BO and produce to D such that  $BO = OD$ .

Step VI: Join BC, CD and AD.

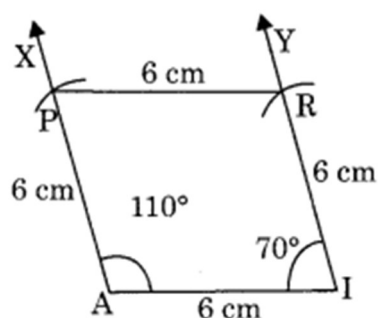
Thus ABCD is the required parallelogram.

9.

Since in a rhombus, all sides are equal, so  $PA = AI = IR = RP = 6$  cm

Also, rhombus is a parallelogram

so, adjacent angle,  $\angle I = 180^\circ - 110^\circ = 70^\circ$



Steps of construction

Step I. Draw  $AI = 6$  cm

Step II. Draw ray  $\overrightarrow{AX}$  such that  $\angle IAX = 110^\circ$  and draw  $\overrightarrow{IY}$  such that  $\angle AIY = 70^\circ$ .

Step III. With A and I as centres and radius 6 cm draw arcs intersecting AX and IY at P and R respectively.

Step IV. Join PR.

Thus, PAIR is the required rhombus.